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KKM mappings in cone b-metric spaces

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ABSTRACT

In this paper we establish some topological properties of the cone b-metric spaces and then improve some recent results about KKM mappings in the setting of a cone b-metric space. We also prove some fixed point existence results for multivalued mappings defined on such spaces.

1. Introduction

Cone metric spaces were introduced in [1]. A similar notion was also considered by Rzepecki in [2]. After carefully defining convergence and completeness in cone metric spaces, the authors in [1] proved some fixed point theorems of contractive mappings. Recently, more fixed point results in cone metric spaces appeared in [3,4]. Topological questions in cone metric spaces were studied in [3] where it was proved that every cone metric space is a first-countable topological space. Hence, continuity is equivalent to sequential continuity and compactness is equivalent to sequential compactness. In this work, with the structure of a cone b-metric space, we shall establish some topological properties of the cone b-metric spaces. We also prove and extend some results of Khamsi and Hussain [5] and illustrate our work in this setting with examples.

2. Basic definitions and results

First, let us start by making some basic definitions.

Let E be a real Banach space. A subset P of E is called a cone if and only if:

(i) P is closed, nonempty and $P \neq \{\theta\}$;
(ii) $a, b \in R$, $a, b \geq 0$, and $x, y \in P$ imply $ax + by \in P$;
(iii) $P \cap (-P) = \{\theta\}$.

Given a cone $P \subset E$, we define a partial ordering $\preceq$ on E with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in intP$ (interior of P). A cone $P \subset E$ is called normal if there is a number $k > 0$ such that for all $x, y \in E$, $\theta \preceq x \preceq y$ implies $\|x\| \leq k\|y\|$. The least positive number satisfying the

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