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SLANT HELICES IN MINKOWSKI SPACE $\mathbf{E}_1^3$

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Abstract. We consider a curve $\alpha = \alpha(s)$ in Minkowski 3-space $\mathbf{E}_1^3$ and denote by $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ the Frenet frame of α. We say that α is a slant helix if there exists a fixed direction U of $\mathbf{E}_1^3$ such that the function $\langle \mathbf{N}(s), U\rangle$ is constant. In this work we give characterizations of slant helices in terms of the curvature and torsion of α. Finally, we discuss the tangent and binormal indicatrices of slant curves, proving that they are helices in $\mathbf{E}_1^3$.

1. Introduction and statement of results

Let $\mathbf{E}_1^3$ be the Minkowski 3-space, that is, $\mathbf{E}_1^3$ is the real vector space $\mathbb{R}^3$ endowed with the standard flat metric

$$\langle , \rangle = dx_1^2 + dx_2^2 - dx_3^2,$$

where (x_1, x_2, x_3) is a rectangular coordinate system of $\mathbf{E}_1^3$. An arbitrary vector $v \in \mathbf{E}_1^3$ is said spacelike if $\langle v, v\rangle > 0$ or $v = 0$, timelike if $\langle v, v\rangle < 0$, and lightlike (or null) if $\langle v, v\rangle = 0$ and $v \neq 0$. The norm (length) of a vector v is given by $|v| = \sqrt{|\langle v, v\rangle|}$.

Given a regular (smooth) curve $\alpha : I \subset \mathbb{R} \to \mathbf{E}_1^3$, we say that α is spacelike (resp. timelike, lightlike) if $\alpha'(t)$ is spacelike (resp. timelike, lightlike) at any $t \in I$, where $\alpha'(t) = d\alpha/dt$. If α is spacelike or timelike we say that α is a non-null curve. In such case, we can reparametrize α by the arc-length $s = s(t)$, that is, $|\alpha'(s)| = 1$. We say then that α is arc-length parametrized. If the curve α is lightlike, the acceleration vector $\alpha''(t)$ must be spacelike for all t. We change the parameter t by $s = s(t)$ in such way that $|\alpha''(s)| = 1$ and we say that α is pseudo arc-length parametrized. In any of the above cases, we say that α is a unit speed curve.

Given a unit speed curve α in Minkowski space $\mathbf{E}_1^3$ it is possible to define a Frenet frame $\{\mathbf{T}(s), \mathbf{N}(s), \mathbf{B}(s)\}$ associated for each point s [5, 7, 10]. Here $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ are the tangent, normal and binormal vector field, respectively. The

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