

# Computational Algorithm for Period Analysis and its Applications to Luminosity Curves of Cepheid Variables

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ABSTRACT. An accurate algorithm has been constructed for determination of the fundamental period  $P_o$  of observed periodic phenomenon. The algorithm is based on the best phase arrangement which produces the minimum polygonal line for the observed data. Numerical application of the algorithm is considered for  $\delta$  Cephei and an accurate period of  $5^d 8^h 55^m 42^s.27$  was obtained together with its mean light curve.

## Introduction

The study of variable stars is extremely important in astronomy, since the category covers many different types associated with critical stages in stellar evolution. The information gained from variable stars has therefore many applications in astronomy. Also study of variable stars as a whole forms one of the major branches of stellar astronomy. Such study is not only an aim in itself, but it can also be used to study, and even solve many problems that faced and still facing astronomers. This concept may be cleared when considering the following points:

1) Several decades ago, astronomers began to use certain group of variable stars as distance indicator, because they are very luminous and their distinctive light variations permit easy and certain identification even at enormous distances<sup>[1]</sup>. Because they are supergiant stars, they can be seen over intergalactic distances. They serve as “Standard Candles” because they are scattered throughout the night sky that serve as mileposts for astronomical surveys of the universe<sup>[2]</sup>.

2) Many groups of variable stars have a particular position in the galaxy and hence they have been used as tools to study the galactic structure.

3) Variable stars play an important role in the field of stellar evolution, since it is known that these stars vary in brightness only at definite stages of their evolution. A good review on this topic, which is based on the study of two Classical Cepheids: Polaris and Y Oph, is given by El-Khateeb<sup>[3]</sup>.

4) The existence of variable stars of the same type in quite different stellar systems allows to believe that evolution of these systems passes through similar stages.

Any observed data of a periodic physical phenomenon, such as the light curve or the radial velocity curve of a variable star, can be classified as deterministic because they can be described by an explicit mathematical relationship. For this reason a periodic phenomenon, or more generally a large scale regular phenomenon, should be accurately observed, so that a theoretical model may be formulated and updated based on further observations.

In this paper, we have constructed an algorithm to determine the fundamental period and the mean light curve of a variable star. Analytical developments of the problem are given in Section 2, while the computational developments are given in Section 3. Numerical applications to  $\delta$  Cephei are given in Section 4.

### Analytical Development

The observation provides  $N$  estimates  $y(k)$ ,  $t(k)$ ;  $k = 1, 2, \dots, N$ , where  $t$  is the time (usually Universal time) and  $y$  the measure of the physical parameters, such as magnitude, radial velocity, radio flux and  $O-C$  residuals, which exhibit the periodic variability (period =  $P_o$ ) that is going to be studied. The used algorithm is based on a sequential scanning of the period<sup>[4]</sup>, with the range  $P_1 \rightarrow P_2$ , which presumably contains the period  $P_o$ , and with a step  $\Delta P$ . The range and the step are input data. The values of the period that will be tried as follow

$$P = P_1 + i \Delta P ; i = 1, 2, \dots \text{ until } P \geq P_2 \quad (1)$$

A second option is followed if the user input  $\Delta P = 0$ , according to

$$P = P_1 + (P_2 - P_1) \left( 1 + \beta - \frac{1}{i} \right) ; i = 1, 2, \dots \text{ until } P \geq P_2 \quad (2)$$

where  $\beta > 0$  is necessary to reach the stop condition for the period series given by Equation (2).

From Equation (2) we note that:

(a) The period series converges to

$$P = P_2 (1 + \beta) - \beta P_1 , \quad \text{for } i \rightarrow \infty \quad (3)$$

with the stopping condition

$$P > P_2 \tag{4}$$

(b) The permissible values of  $\beta$  which allow the limiting value of  $P$  [Equation (3)] to reach the stopping condition of Equation (4) are

$$\beta > 0 \tag{5}$$

(c) It concentrates the values of the period near  $P_2$ , so that this part of the range can be better scanned, and should be chosen whenever the period  $P_o$  is expected to be close but less than  $P_2$ .

Equations (1) or (2) provide  $N_1$  values of the period:  $P_k; k = 1, 2, \dots, N_1$  and therefore  $N_1$  arrangements of the phase array,

$$\phi_j = \{t(j) - t(1)\} / P_k \quad ; \quad j = 1, 2, \dots, N, \quad ; \quad k = 1, 2, \dots, N_1 \tag{6}$$

In order to satisfy the condition  $0 < \phi_j < 1 \quad \forall j = 1, 2, \dots, N$ , the fractional part of each phase should be taken, that is

$$\phi_j = \phi_j - [\phi_j] \tag{7}$$

where  $[x]$  denotes the largest integer  $\leq x$  ;  $x \geq 0$ .

Each phase arrangements correspond to a different length of the polygonal line connecting the points  $(\phi_j, y(j))$  ;

$$R_k \sum_{j=1}^{N-1} \left\{ (\phi_j - \phi_{j+1})^2 + (y(j) - y(j+1))^2 \right\}^{1/2} ; k = 1, 2, \dots, N_1 \tag{8}$$

Illustration of Equation (8) is shown in Fig. 1.

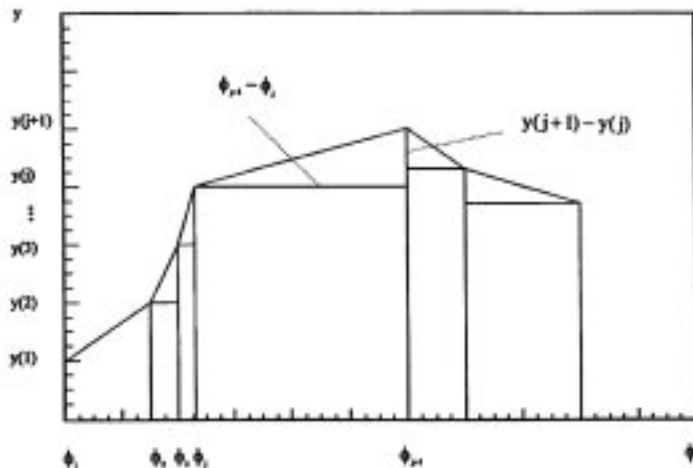


FIG. 1. Illustration of Equation (8).

The period  $P_k$  corresponding to the minimum length of the polygonal line will be chosen as estimate of the  $P_o$ .

### Computational Developments

#### Computational Algorithm (1)

##### Purpose

To arrange arrays  $P$ ,  $T$ ,  $Y$  in the sense of increasing values of the array  $P$ .

##### Input Data

$N$  values of  $P(j)$ ,  $T(j)$ ,  $Y(j)$  ;  $j = 1, 2, \dots, N$ .

##### Computational Sequence

- 1 – For  $k = 1, 2, \dots, N - 1$  , do :
  - Set  $H = P(k)$ ,  $j_1 = k$
  - For  $j = k, k + 1, \dots, N$  do:
    - If  $P(j) \geq H$  Next to  $j$
    - Set  $H = P(j)$  ,  $j_1 = j$
  - Set  $P(j_1) = P(k)$  ,  $P(k) = H$
  - Set  $H = T(j_1)$  ,  $T(j_1) = T(k)$  ,  $T(k) = H$
  - Set  $H = Y(j_1)$  ,  $Y(j_1) = Y(k)$  ,  $Y(k) = H$
- 2 – End.

#### Computational Algorithm (2)

##### Purpose

To arrange arrays  $T$  and  $Y$  in the sense of increasing values of the array  $T$ .

##### Input Data

$N$  values of  $T(j)$  and  $Y(j)$  ;  $j = 1, 2, \dots, N$

##### Computational Sequence

- 1 – For  $k = 1, 2, \dots, N - 1$  , do
  - Set  $H = T(k)$  ,  $j_1 = k$
  - For  $j = k, k + 1, \dots, N$  , do :
    - If  $T(j) \geq H$  Next to  $j$
    - Set  $H = T(j)$  ,  $j_1 = j$
  - Set  $T(j_1) = T(k)$  ,  $T(k) = H$
  - Set  $H = Y(j_1)$  ,  $Y(j_1) = Y(k)$ ,  $Y(k) = H$

2 – End.

### **Computational Algorithm (3)**

#### **Purpose**

To determine  $H$ ,  $R_1$ , and  $I_{\min}$ , where  $H$  is the minimum value of the array  $R$ ,  $R_1$  its maximum value, and  $I_{\min}$  is the order of  $H$  in the array.

#### **Input Data**

$N_1$  values of  $R(j)$  ;  $j = 1, 2, \dots, N_1$ .

#### **Computational Sequence**

- 1 – Set  $H = R(1)$  ,  $j_1 = 1$  ,  $R_1 = R(1)$
- 2 – For  $k = 1, 2, \dots, N_1$  , do :
 

<p style="margin: 0;">If <math>R(K) &gt; H</math> go to 3</p> <p style="margin: 0;">Set <math>H = R(k)</math> , <math>j_1 = K</math></p>
<p style="margin: 0;">3 – If <math>R(k) &lt; R_1</math> Next to <math>K</math></p> <p style="margin: 0;"><math>R_1 = R(k)</math></p>
- 4 –  $I_{\min} = j_1$
- 5 – End.

### **Computational Algorithm (4)**

#### **Purpose**

To determine an estimate of the fundamental period  $P_o$  of periodic phenomena for variable stars using the minimum polygonal line method for observed data, with:

- 1 – Sequential scanning of Equation (1), or
- 2 – Non linear scanning of Equation (2).

#### **Input Data**

- 1 –  $N$  = number of observed data (magnitudes, radial velocity,  $O-C$  residuals, radio fluxes, etc.).
- 2 –  $T(j)$  ,  $Y(j)$  ;  $j = 1, 2, \dots, N \equiv$  observations.
- 3 –  $P_1, P_2$  = range of values of the period that will be investigated.
- 4 –  $IT$  = positive integer, takes the value 1 or 2 such that if  $IT = 1$  the algorithm uses sequential scanning, while for  $IT = 2$  it uses non linear scanning.

5 –  $\alpha$  stands for  $s$  if  $IT = 1$  , and for  $\beta$  if  $IT = 2$ .

### **Computational Sequence**

1 – Use computational algorithm 2 to arrange the arrays T and Y in the sense of increasing values of the T.

2 – If  $IT = 2$  go to step 17

3 – If  $\alpha > (P_2 - P_1)/300$  go to step 5

4 – The value of  $\alpha$  is too small for the given period range. Use larger value of  $\alpha$  and go to step 3

5 – If  $\alpha < (P_2 - P_1)$  go to step 7

6 – The value of  $\alpha$  is too large for the given period range. Use smaller value of  $\alpha$  and go to step 3

7 –  $N_1 = 1$  (Sequential scanning)

8 –  $P_3(N_1) = P_1 + \alpha (N_1 - 1)$

9 – If  $P_3(N_1) > P_2$  go to step 27

10 – For  $j = 1, 2, \dots, N$ , do :

$$\left\{ \begin{array}{l} \phi(j) = (T(j) - T(1))/P_3(N_1) \\ \phi(j) = \phi(j) - [\phi(j)] \end{array} \right.$$

11 – Use computational algorithm 1 to arrange the arrays  $\phi$ , T, and Y in the sense of increasing the phase  $\phi$ .

12 –  $R(N_1) = 0$

13 – For  $j = 1, 2, \dots, N - 1$  , do :

$$\left\{ \begin{array}{l} R(N_1) = R(N_1) + \{[\phi(j) - \phi(j+1)]^2 + [Y(j) - Y(j+1)]\}^{1/2} \end{array} \right.$$

14 – Use computational algorithm 2 to arrange the arrays T( ) and Y( ) in the sense of increasing the values of the T.

15 – If  $N_1 \geq 300$  go to step 27 (Too many iterations ; the loop stops here)

16 –  $N_1 = N_1 + 1$  , go to step 8

17 –  $N_1 = 1$  (The non linear scanning)

18 –  $P_3(N_1) = P_1 + (P_2 - P_1) (1 + \alpha - 1/N_1)$

19 – If  $P_3(N_1) > P_2$  go to step 27

20 – For  $j = 1, 2, \dots, N$  , do :

$$\left\{ \begin{array}{l} \phi(j) = (T(j) - T(1))/P_3(N_1) \\ \phi(j) = \phi(j) - [\phi(j)] \end{array} \right.$$

21 – Use computational algorithm 1 to arrange the arrays  $\phi( )$ , T( ) and Y( ) in the sense of increasing the values of the phase  $\phi$ .

22 –  $R(N_1) = 0$

23 – For  $j = 1, 2, \dots, N - 1$ , do :

$$\left| R(N_1) = R(N_1) + \{ [\phi(j) - \phi(j+1)]^2 + [Y(j) - Y(j+1)]^2 \}^{1/2}$$

24 – Use computational algorithm 2 to arrange the arrays T and Y in the sense of increasing values of the T.

25 – If  $N_1 \geq 300$  go to step 27

26 –  $N_1 = N_1 + 1$ , go to step 18

27 – Use computational algorithm 3 to compute the minimum value H and the maximum value  $R_1$  of the array R, also to compute the order  $I_{\min}$  of the value H.

28 –  $P_o = P_3 (I_{\min})$

29 – End.

### **Computational Algorithm (5)**

#### **Purpose**

To compute approximate value of the period  $P$ .

#### **Input Data**

$$m_j, t_j ; j = 1, 2, 3, \dots, N \text{ and } P_a = [P]$$

#### **Computational Sequence**

1 – From the list of observations, identify the maxima and minima.

2 – Determine the intervals between successive maxima, let their sum be A.

3 – Determine the number of cycles between the intervals of successive maxima, let their sum be  $S_1$ .

4 – Determine the intervals between successive minima, let their sum be B.

5 – Determine the number of cycles between the intervals of successive minima, let their sum be  $S_2$ .

6 – Compute an approximate value of the period P from :

$$P = (A + B) / (S_1 + S_2).$$

7 – End.

### **Computational Algorithm (6)**

#### **Purpose**

To compute the difference,  $\varepsilon = O - C$ , between the observed and calculated times of each maxima.

**Input Data**

- 1 – The observed values of  $m_j, t_j ; j = 1, 2, \dots, N$
- 2 – The period  $P$ .

**Computational Sequence**

- 1 – From the observed values determine the epoch  $E_0$  of the first maximum.
- 2 – Determine the observed date of maximum (O) and the corresponding number of the cycle X.
- 3 – Compute the calculated value of the maxima from  $C = E_0 + XP$ .
- 4 – Compute  $\varepsilon$ , the difference between the observed and calculated times of each maximum from  $\varepsilon = O - C$ .
- 5 – End.

**Computational Algorithm (7)****Purpose**

To compute the coefficients  $a_0$  and  $a_1$  for the least squares fit of the relation  $\varepsilon = a_0 + a_1 X$

**Input Data**

$$e_j, X_j ; j = 1, 2, \dots, n$$

**Computational Sequence**

- 1 – Compute the sums

$$T = \sum_{i=1}^n X_i, \quad S = \sum_{i=1}^n X_i^2, \quad B_1 = \sum_{i=1}^n \varepsilon_i, \quad B_2 = \sum_{i=1}^n \varepsilon_i X_i, \quad W = \sum_{i=1}^n \varepsilon_i^2,$$

- 2 – Compute  $\Delta$  from  $\Delta = n S - T^2$ .
- 3 – Compute  $a_0$  and  $a_1$  from

$$a_0 = (B_1 S - B_2 T) / \Delta, \quad a_1 = (B_2 n - B_1 T) / \Delta.$$

- 4 – End.

**Computational Algorithm (8)****Purpose**

To compute the values of the period  $P$  and the epoch  $E_0$  of the first maximum.



**Input Data**

$$P, E_0, \varepsilon_j, X_j \text{ and } Tol ; j = 1, 2, \dots, n$$

where  $Tol$  is a given tolerance.

**Computational Sequence**

1 – Use algorithm 7 to compute  $a_0$  and  $a_1$ .

2 – If  $(|a_0| \text{ and } |a_1|, Tol)$  go to step 6.

3 – Compute the improved values of  $P$  and  $E_0$  from

$$P = P + a_1, E_0 = E_0 + a_0.$$

4 – With this value of  $P$ , use algorithm 6 to compute new values of  $\varepsilon$ .

5 – Go to step 1.

6 – Consider the last two values of  $P$  as  $P_1$  and  $P_2$ , then use algorithm 4 to find the final improvement of the period  $P$ .

7 – End.

**Computational Algorithm (9)****Purpose**

To compute the phase  $\phi$  of the light curve.

**Input Data**

$$E_0, P, m_j \text{ and } t_j ; j = 1, 2, \dots, N.$$

**Computational Sequence**

1 – Compute  $\forall j = 1, 2, \dots, N, Z_j$  from  $Z_j = (t_j - E_0) / P$ .

2 – Compute the phases  $\Phi_j ; j = 1, 2, \dots, N$  from  $\Phi_j = Z_j - [Z_j]$

3 – End.

**Some Notes**

- a) It is recommended to plot the polygonal length against the period to:
  1. Allow us to identify the presence of secondary minima.
  2. Verify that the selected value of the period corresponds to an absolute minimum of  $R_k ; k = 1, 2, \dots, N - 1$ .
- b) The precision of the algorithm depending on scanning value  $\alpha$ .
- c) In order to increase the precision, the computation may be carried out in two different steps: first, run Algorithm 4 with  $IT = 1$  with  $\alpha$  not too large {for example  $(P_2 - P_1)/200$ } and then run the algorithm with  $IT = 2$  with  $P_2$  very close to, but greater than, the first result.

### Numerical Applications and Results

The visual and photo-visual observations of  $\delta$  Cephei<sup>[5]</sup> are listed in Table 1. To construct the light (luminosity) curve of  $\delta$  Cephei, the following steps are to be performed.

TABLE 1. Visual and photo-visual observations of  $\delta$  Cephei, obtained between September and December, 1979 by E. Schweitzer.

Date	Time	Julian day 2 444 000 +	Comparison	$m_v$	Date	Time	Julian day 2 444 000 +	Comparison	$m_v$
Sept. 16	19.12	133.3007	a6 v 1b	4.2	Oct. 22	18.32	169.2722	a3 v 4b	3.9
16	23.52	133.4944	a6 v 1b	4.2	22	22.15	169.4271	a3 v 4b	3.9
17	19.09	134.2979	a6 v 1b	4.2	23	18.08	170.2556	a5 v 2b	4.1
17	22.55	134.4449	a6 v 1b	4.2	24	17.32	171.2305	v b	4.3
18	19.06	135.2958	a2 v 5b	3.8	24	23.28	171.4778	b1 v	4.4
18	21.20	135.3889	v 2a	<u>3.4</u>	25	17.42	172.2375	a4 v 3b	4.0
18	22.51	135.4521	a1 v 6b	3.7	25	22.55	172.4549	a5 v 3b	4.1
24	19.41	141.3222	v a	3.6	26	17.22	173.2236	v a	3.6
25	19.45	142.3230	a2 v 4b	3.8	26	23.02	173.4597	v a	3.6
26	18.31	143.2715	a2 v 5b	3.8	Nov. 1	17.23	179.2312	v a	3.6
26	20.56	143.3722	a3 v 3b	3.9	2	18.05	180.2535	a3 v 4b	3.9
26	23.48	143.4917	a4 v 2b	4.0	4	19.43	182.3216	v 1b	4.4
27	18.30	144.2708	a5 v 1b	4.2	4	22.00	182.4167	v b	4.3
27	23.57	144.4979	b1 v	4.4	5	17.02	183.2097	a4 v 3b	4.0
28	18.39	145.2771	a4 v 3b	4.0	9	18.11	187.2576	a5 v 1b	4.2
28	19.22	145.3070	a4 v 3b	4.0	9	22.26	187.4348	v b	4.3
29	19.12	146.3000	v a	3.6	11	16.55	189.2049	v a	3.6
30	18.06	147.2535	a1 v 5b	3.7	12	00.33	189.5229	v14a	3.5
30	20.21	147.3479	a1 v 5b	<u>3.7</u>	14	21.53	192.4118	v b	4.3
Oct. 1	01.59	147.5771	a2 v 5b	3.8	16	19.25	194.3091	a2 v 5b	3.8
1	18.20	148.2639	a3 v 4b	3.9	17	01.00	194.5417	v a	3.6
1	21.17	148.3868	a2 v 4b	<u>3.8</u>	17	16.46	195.1987	a1 v 6b	3.7
2	01.46	148.5737	a4 v 3b	4.0	22	16.45	200.1980	v a	3.6
2	18.03	149.2521	a5 v 2b	4.1	28	20.50	206.3680	a3 v 4b	3.9
2	21.51	149.4104	a4 v 3b	<u>4.0</u>	30	17.05	208.2118	a5 v 1b	4.2
2	22.04	149.4195	a5 v 1b	4.2	Dec. 2	16.50	210.2014	a4 v 3b	4.0
3	17.55	150.2465	b1 v	4.4	2	23.00	210.4583	a2 v 5b	3.8
3	20.18	150.3458	v b	<u>4.3</u>	3	17.00	211.2083	v 1a	3.5
6	17.55	153.2465	a3 v 4b	3.9	3	22.30	211.4375	a1 v 5b	3.7
7	18.50	154.2847	a3 v 4b	3.9	4	17.42	212.2375	a4 v 3b	4.0
8	18.35	155.2743	vb	4.3	4	21.55	212.4132	a4 v 3b	4.0
9	18.46	156.2830	a3 v 4b	3.9	7	16.40	215.1945	a5 v 2b	4.1
10	18.34	157.2736	v a	3.6	7	21.33	215.3979	a5 v 2b	4.1
12	17.44	159.2389	a3 v 4b	3.9	11	16.35	219.1910	a5 v 2b	4.1
15	17.23	162.2243	a1 v 5b	3.7	12	22.02	220.4181	v b	4.3
18	18.42	165.2792	a5 v 2b	4.1	14	16.16	222.1778	a2 v 5b	3.8
18	22.00	165.4167	a5 v 1b	4.2	16	17.23	224.2243	a5 v 2b	4.1

TABLE 1. Contd.

Date	Time	Julian day 2 444 000 +	Comparison	$m_v$	Date	Time	Julian day 2 444 000 +	Comparison	$m_v$
19	18.05	166.2535	v b	4.3	18	17.00	226.2083	a2 v 5b	3.8
19	22.42	166.4459	b1 v	4.4	23	18.42	231.2792	a5 v 2b	4.1
20	17.24	167.2250	a3 v 4b	3.9	25	21.21	235.3896	a5 v 2b	4.1
20	21.20	167.3889	a1 v 6b	3.7	30	17.48	238.2417	a2 v 5b	3.8
21	19.17	168.3035	v a	3.6	31	23.18	239.4708	a3 v 4b	3.9
21	23.05	168.4618	v a	3.6					

Note: Photo-visual observations are underlined.

1) Find an approximate value of the period  $P$  by using Algorithm 5, with  $m_j$  and  $t_j$  as given in Table 2 (and shown in Fig. 2) and  $P_a = 5$  days. The output of the algorithm is listed in Table 3 (and shown in Fig. 3). It follows from Table 3 that

$$P + (A + B) / (S_1 + S_2) = (69.8861 + 47.9139) / (13 + 9)$$

so that the approximate value of the period is  $P = 5.354517$  days

TABLE 2. Data for the light curve of a variable star.

Day	m	Day	m	Day	m
0	5.45	30	5.31	60	4.50
5	5.10	35	5.40	65	4.79
10	4.50	40	5.47	70	5.02
15	4.79	45	5.50	75	5.18
20	5.02	50	5.45	80	5.31
25	5.18	55	5.10		

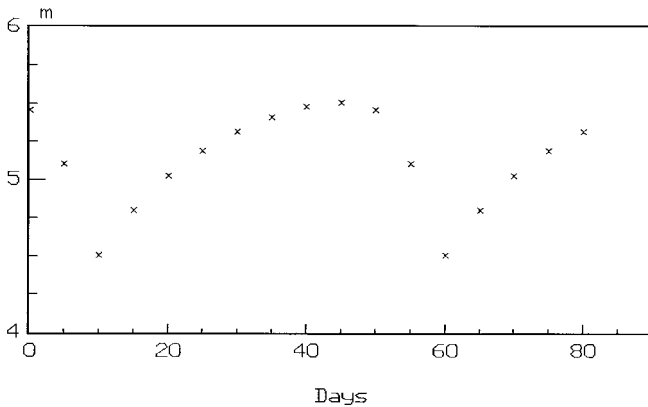


FIG. 2. Data for the light curve of a variable star.

TABLE 3. Approximate value of the period.

Maxima			Minima		
Julian date of observations 2 444 000 +	Length of the cycle (days)	Number of cycles	Julian date of observations 2 444 000 +	Length of the cycle (days)	Number of cycles
141.3222	4.9778	1	144.4979	5.7983	1
146.3000	10.9736	2	150.2962	4.9781	1
157.2736	5.0597	1	155.2743	11.0754	2
162.3333	6.0493	1	166.3497	5.0045	1
168.3826	4.9590	1	171.3542	11.0150	2
173.3416	5.8896	1	182.3692	5.0656	1
179.2312	10.1327	2	187.4348	4.9764	1
189.3639	5.1778	1	192.4118		
194.5417	4.6563	1			
200.1980	11.0103	2			
211.2083					

$$A = 69.8861 \quad S_1 = 3 \quad B = 47.9139 \quad S_2 = 9$$

2) Find the difference,  $\varepsilon = O - C$ , between the observed and calculated times of each maximum by using Algorithm 6, with  $P = 5.354517^d$  and  $E_o = 2444141.3222$ . The output is listed in Table 4 (and shown in Fig. 4).

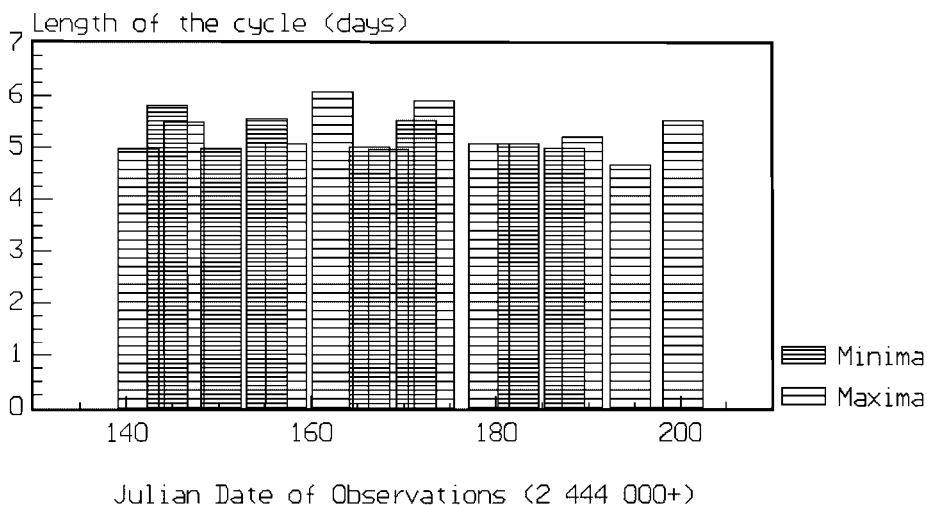
FIG. 3. Approximate period of  $\delta$  Cephei.

TABLE 4. The differences  $\varepsilon = O - C$ .

$X$	Observed date of maximum ( $O$ ) 2 444 000 +	Calculated date of maximum ( $C$ ) 2 444 000 +	$\varepsilon = O - C$
0	141.3222	141.322200	0
1	146.3000	146.676717	-0.376717
3	157.2736	157.385751	-0.112151
4	162.3333	162.740268	-0.406968
5	168.3826	168.094785	+0.287815
6	173.3416	173.449302	-0.107702
7	179.2312	178.803819	-0.427381
9	189.3639	189.512853	-0.148953
10	194.5417	194.867370	-0.325670
11	200.1980	200.221887	-0.023887
13	211.2083	210.930921	+0.277379

3) Find the improved values of the period  $P$  and the epoch  $E_o$  of the first maximum by using Algorithm 8 as follows:

a) With  $P = 5.354517^d$ ;  $E_o = 2444141.3222$ ;  $\varepsilon_j, X_j, j = 1, 2, \dots, 11$  given from Table 4, and  $Tol = 10^{-5}$ , Algorithm 7 gives  $a_0 = -0.168224$ ,  $a_1 = +0.019435$ .

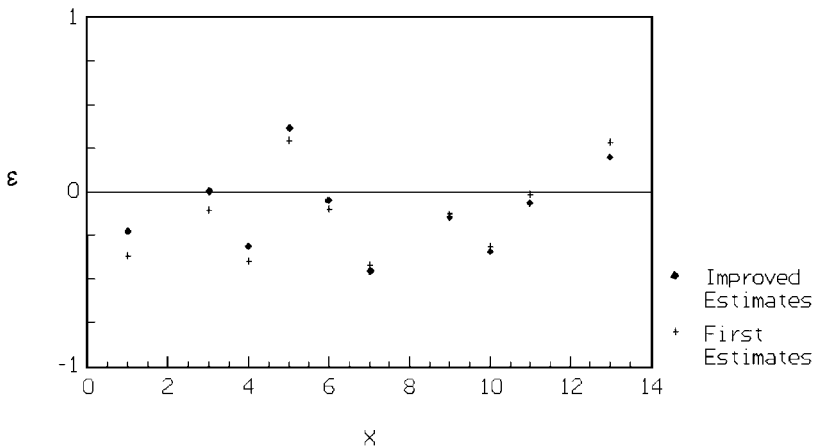


FIG. 4. The differences  $\varepsilon = O - C$ .

b) Since  $|a_0|$  and  $|a_1| > Tol$ , then

$$P = P + a_1 = 5.373952 \quad , \quad E_o = E_o + a_0 = 2444141.153976.$$

c) With this value of  $P$ , Algorithm 6 gives new values of  $\varepsilon$  which are listed in Table 5 (and shown in Fig. 4).

TABLE 5. New values of differences  $\varepsilon = O - C$ .

$X$	Observed date of maximum ( $O$ ) 2 444 000 +	Calculated date of maximum ( $C$ ) 2 444 000 +	$\varepsilon = O - C$
0	141.3222	141.1539	0.1682
1	146.3000	146.5279	- 0.2279
3	157.2736	157.2758	- 0.0022
4	162.3333	162.6498	- 0.3165
5	168.3826	168.0237	0.3589
6	173.3416	173.3977	- 0.0561
7	179.2312	178.7716	- 0.4596
9	189.3639	189.5195	- 0.1556
10	194.5417	194.8935	- 0.3518
11	200.1980	200.2674	- 0.0694
13	211.2083	211.0154	+ 0.1929

d) With the  $X^s$  and  $\varepsilon^s$  from Table 5, Algorithm 7 gives

$$a_0 = 4.271397 \times 10^{-11} \quad , \quad a_1 = - 0.6075956 \times 10^{-14}.$$

e) Since  $|a_0|$  and  $|a_1| < Tol$ , then consider  $P_1$  and  $P_2$  as

$$P_1 = 5.354517^d \quad , \quad P_2 = 5.3739517^d$$

as the input values of Algorithm 4, and we get for the period the final value

$$P = 5.372017^d = 5^d 8^h 55^m 42^s.27$$

Table 6 gives values of the periods and the polygonal length. From this table we see that the minimum value of the polygonal length is 7.987093127233910 which corresponds to the period  $P = 5.372017^d$ , which confirmed the above result of the final period. Values of the color, luminosity, mass, and radius of the star  $\delta$  Cephei can therefore be computed from the well-known relation<sup>[6-9]</sup>.

4) Compute the phases using Algorithm 9 and we get Table 7 between phases  $\phi$  and the apparent magnitudes  $m$ . Finally, the required luminosity curve of  $\delta$  Cephei is given in Fig. 5; the SPLINE procedure is used to obtain the best fit to the light curve.

In concluding this paper, an accurate algorithm has been constructed for period determination of periodic phenomenon. The algorithm was applied for  $\delta$  Cephei in order to determine its period and mean light curve.

TABLE 6. Periods against polygonal length.

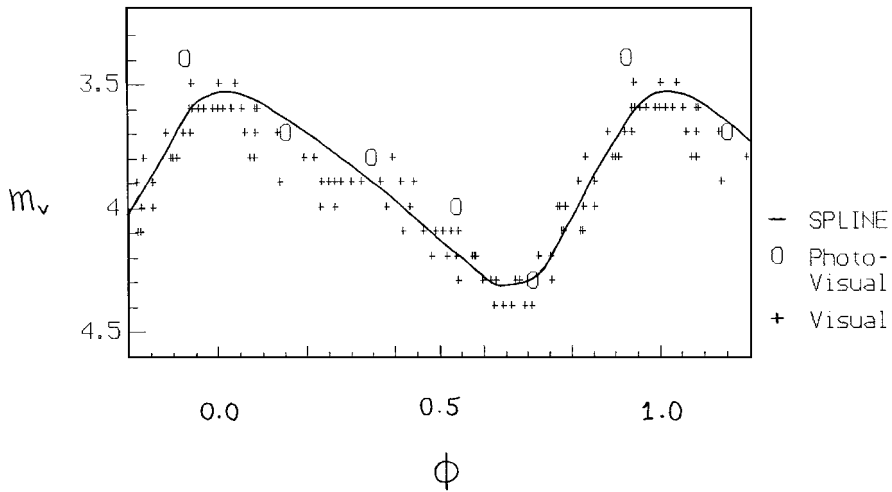
<i>P</i>	<i>R</i>	<i>P</i>	<i>R</i>
9.3539949	7.98709312723391	5.3640170	9.14592725084059
5.3545170	9.14204661547270	5.3645170	9.14508997310208
5.3550170	9.14504987123653	5.3650170	9.14185571943845
5.3555170	9.14927487739509	5.3655170	9.33878846355305
5.3560170	9.34374886562902	5.3660170	9.33971323162058
5.3565170	9.16133834392907	5.3665170	9.34093136437293
5.3570170	9.14714647145330	5.3670170	9.15619710929421
5.3575170	9.14766210044817	5.3675170	8.74177743519474
5.3580170	8.96811903601509	5.3680170	8.74111067783573
5.3585170	8.96543689697828	5.3685170	8.83711771969924
5.3590170	8.96358932988842	5.3690170	8.83778109592244
5.3595170	8.96678962938762	5.3695170	8.83923796704616
5.3600170	9.15885864489083	5.3700170	8.34926856886708
5.3605170	9.15792073823789	5.3705170	8.35123335639887
5.3610170	9.35399485957666	5.3710170	8.36753964520533
5.3615170	9.35373447388736	5.3715170	8.37014677692709
5.3620170	9.35035398911963	5.3720170	7.98709312723391
5.3625170	9.16164923116921	5.3725170	7.98951129474225
5.3630170	9.17408883830942	5.3730170	8.19236371595891
5.3635170	9.14702614916558	5.3735170	8.16332238140989

TABLE 7. The phases and apparent magnitudes for  $\delta$  Cephei.

$\phi$	<i>m</i>	$\phi$	<i>m</i>	$\phi$	<i>m</i>
-.4619	4.2	0.6285	4.3	0.9947	3.6
-.4258	4.2	0.8163	3.9	0.0039	3.5
-.2763	4.2	0.0007	3.6	0.5416	4.3
-.2489	4.2	0.3665	3.9	0.8948	3.8
-0.905	3.8	0.9222	3.7	0.9381	3.6
-.0732	3.4	0.9425	3.5	0.0604	3.7
-.0614	3.7	0.4909	4.1	0.9910	3.6
0.0313	3.6	0.5165	4.2	0.1396	3.9
0.2176	3.8	0.6723	4.3	0.4828	4.2
0.3942	3.8	0.7081	4.4	0.8532	4.0
0.4129	3.9	0.8531	3.9	0.9010	3.8
0.4352	4.0	0.8836	3.7	0.0406	3.5
0.5802	4.2	0.0539	3.6	0.0833	3.7
0.6225	4.4	0.0833	3.6	0.2322	4.0
0.7675	4.0	0.2342	3.9	0.2649	4.0

TABLE 7. Cont.

$\phi$	$m$	$\phi$	$m$	$\phi$	$m$
0.7731	4.0	0.2630	3.9	0.7826	4.1
0.9579	3.6	0.4173	4.1	0.8205	4.1
0.1354	3.7	0.5987	4.3	0.5266	4.1
0.1530	3.7	0.6448	4.4	0.7550	4.3
0.1957	3.8	0.7862	4.0	0.0826	3.8
0.3235	3.5	0.8267	4.1	0.4635	4.1
0.3464	3.8	0.9698	3.6	0.8328	3.8
0.3812	4.0	0.0137	3.6	0.7768	4.1
0.5075	4.1	0.0880	3.6	0.5419	4.1
0.5369	4.0	0.2784	3.9	0.0729	3.8
0.5386	4.2	0.6633	4.4	0.3016	3.9
0.6926	4.4	0.6810	4.3		
0.7111	4.3	0.8287	4.0		
0.2510	3.9	0.5822	4.2		
0.4443	3.9	0.6152	4.3		

FIG. 5. Luminosity curve of  $\delta$  Cephei.

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## برنامج حسابي لتحليل دورة التغيير في النجوم القيفاوية وتطبيقاته على منحنيات الاستضاءة لهذه النجوم

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جدة - المملكة العربية السعودية

المستخلص . تم تصميم برنامج حسابي دقيق لتعيين دورة التغيير للظواهر الدورية على أفضل واجهة مُرتبة حتى يُعطى أصغر قيمة لأفضل شكل يمكنه التعبير عن البيانات المرصودة. أُجريت تطبيقات عديدة بهذا البرنامج على النجم القيفاوي المتغير ( دلتا ) ، وحصلنا على قيمة دقيقة لدورة التغيير ، وهي ٥ أيام ، ٨ ساعات ، ٥٥ دقيقة ، ٢٧ ، ٤٢ ثانية . كما حصلنا على المنحنى الضوئي المتوسط لهذا النجم .