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Three-Level Atom and Two Modes: The Ladder Configuration

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ABSTRACT. A model is presented to discuss the interaction between 2 modes and a 3-level atom in the cascade type. There is a detunning parameter. The model is solved with the interaction terms in the Hamiltonian assisted through functions of the photon no·s in the two modes. The probability distribution functions are calculated and characteristic functions as well as mean values for the photon no·s and their powers and the occupation numbers in the atomic levels are computed. The multiphoton processes are discussed by specifying the functions of the photon numbers.

Introduction

The problem of a 3-level atom that interacts with 2 modes has been given vigorous investigations recently[1-5] through a full quantum mechanical treatment. Most of the investigations[1-4] treat the case of what is called the "Λ" type where the two lower levels are connected to the upper level. The "V" type has also been studied recently[5] where the upper two levels are connected to the lower level. As far as we are aware, the "cascade" type has not been investigated generally apart from the special case of a one-mode, equal energy spacing and two equal coupling constants studied by Sukumar & Buck[6]. In this article, we present a generalized model for the 3-level atom and 2 modes in interaction. The three states of the atom have energies ω_1 , ω_2 and ω_3 where $\omega_1 > \omega_2 > \omega_3$. The interaction between levels 1 and 2 is affected by the mode 1 of energy Ω_1, while the mode 2 of energy Ω_2 connects the two levels 2 and 3. The model presented here contains a detunning parameter, Δ, *i.e.* the transitions are not in exact resonance with the photons energies. The scheme for the interaction is presented in Fig. 1. The interactions are assisted by the functions f_1 $(\hat{n}_1 , \hat{n}_2)$ and f_2 $(\hat{n}_1, \hat{n}_2)$ of the photon-numbers $\hat{n}_1$, $\hat{n}_2$ in the two modes. These functions are introduced in a formal way. When multiphoton processes are discussed, these functions are specified as we show later.

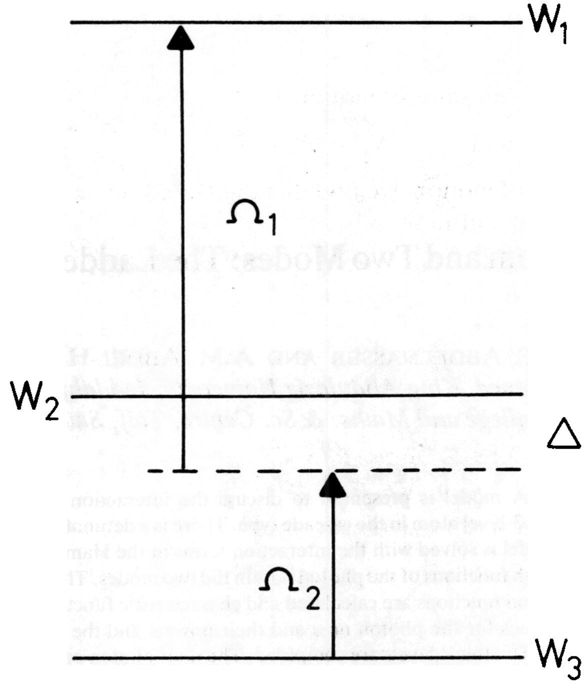

FIG. 1

Description of the System

The Hamiltonian for such a system can be written in the following form

$$H = \sum_{\alpha=1}^{3} \omega_\alpha \hat{S}_{\alpha\alpha} + \sum_{j=1}^{2} \Omega_j \hat{a}_j^+ \hat{a}_j + \sum_{j=1}^{2} \lambda_j (\hat{S}_{j,\,j+1} \hat{R}_j + \hat{R}_j^+ \hat{S}_{j+,\,j}) \qquad (1)$$

where $\hat{S}_{\alpha\beta}$ are the generators of the U (3) group, see Li and Zhu[7].

and $\hat{R}_i = \hat{a}_i f_i (\hat{n}_1, \hat{n}_2)$ with f_i a real function of the photon number operators $(\hat{n}_i = \hat{a}_i^+ \hat{a}_i)$ of the two modes. The operators $\hat{S}_{\alpha\beta}$ and $\hat{R}_i$ are supposed to commute with each other.

They satisfy the following equations :

$$[\hat{R}_i , \hat{n}_i] = \hat{R}_i , \ [\hat{R}_i^+ , \hat{n}_i] = -\hat{R}_i^+ , \ [\hat{S}_{\alpha\beta}\ \hat{S}_{\mu\nu}] = \hat{S}_{\alpha\nu}\, \delta_{\beta\mu} - \hat{S}_{\mu\beta}\, \delta_{\alpha\nu}$$

$$[\hat{R}_1, \hat{R}_1^+] = (\hat{n}_1 + 1)\, f_1^2(\hat{n}_1 + 1, \hat{n}_2) - \hat{n}_1 f_1^2(\hat{n}_1, \hat{n}_2) \qquad (2)$$

with a similar relation for $[\hat{R}_2, \hat{R}_2^+]$, where $\hat{a}_i$ and $\hat{a}_i^+$ are the boson operators that introduce the mode i, and they satisfy $[\hat{a}_i, \hat{a}_j^+] = \delta_{ij}$.

Under the condition

$$\Delta = \omega_j - \omega_{j+1} - \Omega_j \qquad j = 1,\ 2 \tag{3}$$

We find the following constants of motion

$$\hat{N}_1 = \hat{n}_1 + \hat{S}_{11} \quad \text{and} \quad \hat{N}_2 = \hat{n}_2 - \hat{S}_{33} \tag{4}$$

Using these constants of motion, we find that the Hamiltonian (1) breaks up into the following commuting quantities

$$H = \hat{C} + \hat{D} \tag{5}$$

where

$$\hat{D} = (\omega_2 - \Delta)\, I + \Omega_1 \hat{N}_1 + \Omega_2 \hat{N}_2$$

and

$$\hat{C} = \Delta \hat{S}_{22} + \sum_{j=1}^{2} \lambda_j (\hat{S}_{i,\,j+1} \hat{R}_j + \hat{R}_j^+ \hat{S}_{j+1,\,j}) \tag{5b}$$

They commute with each other and hence with H.

We look now for the evolution operator $U(t\,,\,o)$ which is written in the following form :

$$U(t\,,\,o) = \exp(-\,iHt) = \exp(-\,i\hat{D}t)\exp(-\,i\hat{C}t) \tag{6}$$

using the commutation properties of $\hat{C}$ and $\hat{D}$[1,5,7]. Once this operator is calculated, the time evolution for any operator $\hat{O}$ can be computed through the formula

$$\hat{O}(t) = \hat{U}(t\,,\,o)\,\hat{O}(o)\,\hat{U}^+(t\,,\,o) \tag{7}$$

with $\hat{U}^+$ the hermitian conjugate of U.

It is observed that the term $\{\exp(-\,i\hat{D}t)\}$ gives only phase factors which are irrelevant to the expectation values that will be calculated later. The term $\{\exp(-\,iCt)\}$ is expressed in the following matrix form :

$$\exp(-\,iCt) = e^{\frac{i\Delta t}{2}} \begin{cases} e^{\frac{i\Delta t}{2}} + \lambda_1 \hat{R}_1 \hat{A}_1 \lambda_1 \hat{R}_1^+ - i\lambda_1 \hat{R}_1 \dfrac{\sin \mu_1 t}{\mu_1} \lambda_1 \hat{R}_1 \hat{A}_1 \lambda_2 \hat{R}_2 \\[2ex] -i\,\dfrac{\sin \mu_1 t}{\mu_1} \lambda_1 R_1^+ \qquad \cos t\,\mu_1 - \dfrac{i\Delta}{2\mu_1} \sin \mu_1 t - i\,\dfrac{\sin \mu_1 t}{\mu_1} \lambda_2 \hat{R}_2 \\[2ex] \lambda_2 \hat{R}_2^+ \hat{A}_1 \lambda_1 \hat{R}_1^+ - i\lambda_2 \hat{R}_2^+ \dfrac{\sin \mu_1 t}{\mu_1}\, e^{\frac{i\Delta t}{2}} + \lambda_2 \hat{R}_2^+ \hat{A}_1 \lambda_1 \hat{R}_2 \end{cases} \tag{8}$$

where

$$\hat{A}_r = \frac{1}{\nu_-} [\cos \mu_r t - e^{\frac{i\Delta t}{2}} + \frac{i\Delta}{2\mu_r} \sin \mu_r t]\ ,\ (r = 0, 1, 2)$$

with $\mu_r^2 = \nu_r + \frac{\Delta^2}{4}$

while

$$\nu_0 = \lambda_1^2 n_1 f_1^2(n_1 , n_2) + \lambda_2^2 f_2^2(n_1, n_2)\, n_2$$

$$\nu_1 = \lambda_1^2 n_1 f_1^2(n_1, n_2) + \lambda_2^2 (n_2 + 1) f_2^2(n_1 , n_2 + 1)$$

and

$$\nu_2 = \lambda_1^2 (n_1 + 1) f_1^2 (n_1 + 1, n_2) + \lambda_2^2 (n_2 + 1) f_2^2 (n_1, n_2 + 1) \tag{9c}$$

We note here the differences between the form (8) in this case and the case of the "Λ type" discussed by Li and Bei[1] or the "*V* type" of Obada and Abdel Hafez[5].

We suppose the density operator of the system $\hat{\rho}(t)$ to be initially :

$$\hat{\rho}(O) = \hat{\rho}_A(o) \otimes \hat{\rho}_F(0) \tag{10}$$

which is the product of the density operator of the atomic system $\hat{\rho}_A(o)$ are the fields $\hat{\rho}_F(o)$. For $t > o$ the density operator for the fields $\hat{\rho}_F(t)$ and the probability distribution function $P(n_1 , n_2 , t)$ for finding n_i photons in the *ith* mode are given by :

$$\hat{\rho}_F(t) = Tr_A \hat{\rho}(t) \ , \ P(n_1 , n_2 , t) = < n_1 , n_2 | \hat{\rho}_F(t) | n_1 , n_2 > \tag{11}$$

Statistical Aspects

We turn our attention now to calculating these quantities in the general formalism for the atom being in one of its states, then giving the time evolution for the photons numbers expectation values and the occupation numbers for the atomic states.

A. *Atom in Its Ground State of Energy*

In this case the density operator $\hat{\rho}(o)$ and the probability distribution function P are given by :

$$\hat{\rho}^{gr}(0) = \hat{\rho}_F(0)\, S_{33} \tag{12a}$$

and

$$P_\Delta^{gr}(n_1 , n_2 , t) = \lambda_1^2 (n_1 + 1) f_1^2 \quad (n_1 + 1 , n_2)\, \lambda_2^2 (n_2 + 1)$$
$$f_2^2 (n_1 + 1 , n_2 + 1) \quad |A_2|^2\, P(n_1 + 1 , n_1 + 1)$$
$$+ \lambda_2^2 (n_2 + 1) f_2^2 (n_1 , n_2 + 1) \frac{\sin^2 \mu_1 t}{\mu_1^2} P(n_1 , n_2 + 1)$$
$$+ | [e^{\frac{i\Delta t}{2}} + \lambda_2^2 n_2 f_2^2 (n_1 , n_2)\, A_0] |^2 P(n_1 , n) \tag{12b}$$

With $P(n_1 , n_2)$ standing for the probability distribution function for the fields at time $t = 0$.

The characteristic functions $\chi(\beta_1, \beta_2)$ and $\chi_j(\beta)$ are defined by

$$\chi(\beta_1, \beta_2) = \sum_{n_1, n_2} \exp(i\beta_1 n_1 + i\beta_2 n_2) P(n_1, n_2, t)$$

and

$$\chi_i(\beta) = \sum_{n_1, n_2} \exp(i\beta n_j) P(n_1, n_2 t)$$

From these functions the expectation value for any moment of the photon number in any mode $< n_1^{k_1}(t)\, n_2^{k_2}(t) >$ can be easily calculated as the appropriate partial differentiation.

In this case of the atom starting in its ground state, we find

$$\chi^{gr}(\beta_1, \beta_2) = \sum_{n_1, n_2} \exp(i\beta_1 n_1 + i\beta_2 n_2) P(n_1, n_2) [1 + (e^{i\beta_2} - 1) \lambda_2^2 n_2 f_2^2$$

$$(n_1, n_2) \frac{\sin^2 \mu_0 t}{\mu_0^2} + |A_0|^2 (e^{i\beta_1 - i\beta_2} - 1)$$

$$\lambda_1^2 n_1 f_1^2(n_1, n_2 - 1) \lambda_2^2 n_2 f_1^2(n_1, n_2)$$

$$\chi_1^{gr}(\beta) = \chi_\Delta^{gr}(\beta, 0), \quad \chi_2^{gr}(\beta) = \chi_\Delta^{gr}(O, \beta) \qquad (14b)$$

While the different expectation values for the photon no$\cdot$*s* and their 2*nd* powers as well as the occupation no$\cdot$*s* in the atomic levels, which are calculated from the constants (4), are given by :

$$< n_1(t) >_\Delta^{gr} = \bar{n}_1 - \sum_{n_1, n_2} P(n_1 \; n_2) \lambda_1^2 n_1 f_1^2(n_1, n_2 - 1) \lambda_2^2 n_2 f_2^2(n_1, n_2) |A_0|^2$$

$$< n_2(t) >_\Delta^{gr} = \bar{n}_2 - \sum_{n_1, n_2} P(n_1, n_2) \quad \lambda_2^2 n_2 f_2^2(n_1, n_2)$$

$$\lambda_1^2 n_1 f_1^2(n_1, n_2 - 1) |A_0|_+^2 \frac{\sin^2 \mu_0 t}{\mu_0^2}$$

and

$$< n_1^2(t) >_\Delta^{gr} = \bar{n}_1^2 - (< n_1(t) >_\Delta^{gr} \; \bar{n}_1) - 2 \sum_{n_1, n_2} P(n_1, n_2) \lambda_1^2 n_1^2 f_1^2$$

$$(n_1, n_2 - 1) \lambda_2^2 n_2 \times \quad f_2^2(n_1, n_2) |A_0|^2$$

$$< n_2^2(t) >_\Delta^{gr} = \bar{n}_2^2 - (< n_2(t) >_\Delta^{gr} \; n_2) - 2 \sum_{n_1, n_2} P(n_1, n_2) \lambda_2^2 n_2^2 f_2^2(n \; n_2)$$

$$\times \left[\lambda_1^2 n_1 f_1^2 (n_1 , n_2 - 1) \, |A_0|^2 + \frac{\sin^2 \mu_0 t}{\mu_0^2} \right]$$

$$< n_1 (t)\, n_2(t) >_{\Delta}^{gr} = \overline{n_1 , n_2} - \sum_{n_1 , n_2} P(n_1 , n_2)\, \lambda_2^2 n_2 f_2^2(n_1 , n_2)$$

$$\times \left[n_1 \frac{\sin^2 \mu_0 t}{\mu_0^2} + (n_1 + n_2 - 1)\, \lambda_1^2 n_1 f_1^2(n_1 , n_2 - 1)\, |A_0|^2 \right]$$

While the atomic occupation numbers are given by

$$< S_{11}(t) >_{\Delta}^{gr} = \bar{n}_1 - < n_1(t) >_{\Delta}^{gr}$$

$$\sum_{n_1 , n_2} P(n_1 , n_2)\, \lambda_1^2 n_1 f_1^2 (n_1 , n_2 - 1)\, \lambda_2^2 n_2 f_2^2$$

$$(n_1 , n_2)\, |A_0|^2 \tag{16a}$$

$$< S_{33}(t) >_{\Delta}^{gr} = < n_2(t) >_{\Delta}^{gr} - \bar{n}_2 + 1$$

$$= 1 - \sum_{n_2 , n_1} P(n_1 , n_2)\, \lambda_2^2 n_2 f_2^2 (n_1 , n_2) \left[\lambda_1^2 n_1 f_1^2 \right.$$

$$\left. (n_1 , n_2 - 1)\, |A_0|^2 \frac{\sin^2 \mu_0 t}{\mu_0^2} \right]$$

and

$$< S_{22} > = 1 - < S_{11} > - < S_{33} > \tag{16c}$$

where $\bar{\alpha}$ is the initial expectation value of $\hat{\alpha}$.

It is observed that the photons no·s in the two modes as well as the occupation no·s in the 2 upper levels, do not exceed their initial values. This means that if the two modes were initially in vacuum states the system never develops. Also, it is marked that the quantities in (15) & (16) depend on periodic functions of the time, therefore, collapses and revivals are expected as in the other types[1].

B. Atom Starts in Its Intermediate State of Energy (ω_2)

The initial form for the density operation is given by:

$$\hat{\rho}^{in} (0) = \hat{\rho}_F (0)\, \hat{S}_{22}$$

The probability distribution function is thus given by :

$$P_{\Delta}^{in} (n_1 , n_2 t) = \lambda_1^2 (n_1 + 1) f_1^2 (n_1 + 1 , n_2) \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1 + 1 , n_2)$$

$$+ \ (\cos^2 \mu_1 t \ + \ \frac{\Delta^2}{4\,\mu_1^2} \sin^2 \mu_1 t) \ P(n_1 \ , n_2)$$

$$+ \ \lambda_2^2 n_2 f_2^2 \,(n_1 \ , n_2) \ \frac{\sin^2 \mu_0 t}{\mu_0^1} \ P(n_1 \ , n_2 - 1) \tag{17b}$$

Using this function, the statistical quantities calculated above are given by the following formulae in this case

$$\chi^{in} \,(\beta_1 \ , \beta_2) \ = \ \sum_{n_1 \ , n_2} \ \exp \,(i \ \beta_1 n_1 \ + \ i\beta_2 n_2)$$

$$P(n_1 \ , n_2) \ [\ 1 \ + \ (e^{-i\beta_1} - 1) \ \lambda_1^2 n_1 \, f_1^2(n_1 \ , n_1)$$

$$+ \ (e^{i\beta_2} - 1) \ \lambda_2^2 (n_2 \ + \ 1) \ f_2^2(n_1 \ , n_2 \ + \ 1) \ \frac{\sin^2 \mu_1 t}{\mu_1^2} \ ]$$

$$< n_1(t) >_{\Delta}^{in} = \bar{n}_1 - \sum_{n_1 \ , n_2} P(n_1 \ , n_2) \ \lambda_1^2 n_1 \, f_1^2(n_1 \ , n_2) \ \frac{\sin^2 \mu_1 t}{\mu_1^2} \tag{19a}$$

$$< n_2(t) >_{\Delta}^{in} = \bar{n}_2 + \sum_{n_1 \ , n_2} P(n_1 \ , n_2) \ \lambda_2^2 (n_2 \ + \ 1) \, f_2^2 \,(n_1 \ , n_2 \ + \ 1) \ \frac{\sin^2 \mu_1 t}{\mu_1^2}$$

$$< n_1^2(t) >_{\Delta}^{in} = \bar{n}_1^2 - (< n_1(t) >_{\Delta}^{in} - \bar{n}_1) - 2 \sum_{n_1 \ , n_2} P(n_1 \ , n_2)$$

$$[\ \lambda_1 n_1 f_1 \,(n_1 \ , n_2) \ \frac{\sin \mu_1 t}{\mu} \ ]^2$$

$$< n_2^2(t) >_{\Delta}^{in} = \bar{n}_2^2 - (< n_2(t) >_{\Delta}^{in} - \bar{n}_2) - 2 \sum_{n_2 \ , n_1} P(n_1 \ , n_2)$$

$$[\ \lambda_2 \,(n_2 \ + \ 1) \, f_2 \,(n_1 \ , n_2 \ + \ 1) \ \frac{\sin \mu_1 t}{\mu_1} \ ]^2$$

$$< n_1(t) \ n_2(t) >_{\Delta}^{in} = \overline{n_1 n_2} - \sum_{n_1 \ , n_2} P(n_1 \ , n_2) \ [\lambda_1^2 n_1 n_2 \, f_1^2 \,(n_1 \ , n_2)$$

$$+ \ \lambda_2^2 n_1 (n_2 \ + \ 1) \, f_2^2(n_1 \ , n_2 \ + \ 1 \ \times \ \frac{\sin^2 \mu_1 t}{\mu_1^2}$$

The atomic occupation numbers are obtained as follows :

$$< S_{11}(t) >_{\Delta}^{in} \ = \ \bar{n}_1(t) >_{\Delta}^{in}$$

$$\sum_{n_1 \ , n_2} P(n_1 \ , n_2) \ \lambda_1^2 \ n_1 f_1^2 \,(n_1 \ , n_2) \ \frac{\sin^2 \mu_1 t}{\mu_1^2}$$

$$< S_{33}(t) >_{\Delta}^{in} = < n_2(t) >_{\Delta}^{in} - \bar{n}_2$$

$$\sum_{n_2, n_1} P(n_1, n_2) \lambda_2^2 (n_2 + 1) f_2^2 (n_1, n_2 + 1) \frac{\sin^2 \mu_1 t}{\mu_1^2}$$

and $< S_{22} >$ is calculated from (16.c) with $< S_{11} >$, $< S_{33} >$ given by (20a,b).

From these formulae it is noted that the mean *no*· of photons in the *2nd* mode, and the occupation numbers in the ground state (ω_3) and the upper (ω_1) levels are never less than their initial values, while the mean photon number in mode 1 and occupation number in the originally occupied level (ω_2) never exceed their initial values. Thus, if the two modes start from vacuum states, mode 1, which connects the intermediate state that was initially occupied with the upper level, stays in vacuum, and also the upper level (of energy ω_1) stays unoccupied, while mode 2 has a mean value photon *no*· that develops with time.

C. *Atom Initially in Its Upper Level* (ω_1)

For this case, the initial value for the density operator is given by :

$$\hat{\rho}^u (0) = \hat{\rho}_F (0) S_{11} \tag{21a}$$

While the probability distribution function is found to be

$$P_{\Delta}^u (n_1, n_2, t) = | e^{\frac{i\Delta t}{2}} + \lambda_1^2(n_1 + 1) f_1^2(n + 1, n_2) A_2 |^2 P(n_1, n_2)$$
$$+ \lambda_1^2 n_1 f_1^2(n_1, n_2) \frac{\sin^2 \mu_1 t}{\mu_1^2} P(n_1 - 1, n_2) P(n_1 - 1, n_2)$$
$$+ \lambda_1^2 n_1 f_1^2(n_1, n_2 - 1) \lambda_2^2 n_2 f_2^2 (n_1, n_2) |A_0|^2 P(n_1 - 1, n_2 - 1) \tag{21b}$$

The different statistical quantities are given by the following:

For the characteristic functions

$$\chi_{\Delta}^u (\beta_1, \beta_2) = \sum_{n_1, n_2} \exp (i\beta_1 n_1 + i\beta_2 n_2) P(n_1, n_2) [1 + (e^{i\beta_1} - 1)$$
$$\lambda_1^2(n_1 + 1) f_1^2(n_2 + 1, n_2)$$
$$\times \frac{\sin^2 \mu_2 t}{\mu_2^2} + (e^{i\beta_1 + i\beta_2} - 1) \lambda_1^2(n_1 + 1) f_1^2(n_1 + 1, n_2)$$
$$\lambda_2^2(n_2 + 1) f_2^2(n_1 + 1, n_2 + 1) |A_2|^2]$$

The mean values for the photon no·s are

$$< n_1(t) >^u_\Delta = \bar{n}_1 + \sum_{n_1, n_2} P(n_1 , n_2) \lambda_1^2(n_1 + 1) f_1^2(n_1 + 1 , n_2) [\frac{\sin^2 \mu_2 t}{\mu_2^2}$$

$$+ \lambda_2^2 (n_2 + 1) f_2^2(n_1 + 1 , n_2 + 1) |A_2|^2$$

$$< n_2(t) >^u_\Delta = \bar{n}_2 + \sum_{n_2 , n_1} P(n_1 , n_2) \lambda_1^2(n_1 + 1) f_1^2(n_1 + 1 , n_2)$$

$$\lambda_2^2(n_1 + 1 , n_2 + 1) |A_2|^2$$

$$< n_1^2(t) >^u_\Delta = \overline{n_1^2} - (< n_1(t) >^u_\Delta - \bar{n}_1)$$

$$+ 2 \sum_{n_1 , n_2} P(n_1 , n_2) \lambda_1^2(n_1 + 1)^2 f_1^2(n_1 + 1 , n_2)$$

$$\times \frac{\sin^2 \mu_2 t}{\mu_1^2} + \lambda_2^2(n_2 + 1) f_2^2(n_1 + 1 , n_2 + 1) |A_2|^2]$$

$$< n_2^2(t) >^u_\Delta = \overline{n_2^2} - (< n_2(t) >^u_\Delta - \bar{n}_2)$$

$$+ 2 \sum_{n_2 , n_1} P(n_1 , n_2) \lambda_1^2(n_1 + 1) f_1^2(n_1 + 1 , n_2)$$

$$\lambda_2^2(n_2 + 1))^2 f_2^2(n_1 + 1 , n_2 + 1) |A_2|^2$$

$$< n_1(t)\, n_2(t) >^u_\Delta = \overline{n_1 n_2} + \sum_{n_1 , n_2} P(n_1 , n_2) \lambda_1^2(n_1 + 1)$$

$$f_1^2(n_1 + 1 , n_2) [n_2 \frac{\sin^2 \mu_2 t}{\mu_2^2}$$

$$+ (n_1 + n_2 + 1) \lambda_2^2(n_2 + 1) f_2^2(n_1 + 1 , n_2 + 1) |A|^2]$$

For the atomic occupation nos we have

$$< S_{11}(t) >^u_\Delta = \bar{n}_1 - < n_1(t) >^u_\Delta$$

$$= 1 - \sum_{n_2} P(n_1 , n_2) \lambda_1^2(n_1 + 1) f_1^2(n_1 + 1 , n_2) \frac{\sin^2 \mu_2 t}{\mu_2^2} +$$

$$+ \lambda_2^2(n_2 + 1) f_2^2 (n_1 + 1 , n_2 + 1) |A_2|^2 \tag{24a}$$

and

$$< S_{33}(t) >^u_\Delta = < n_2(t) >^u_\Delta - \bar{n}_2$$

$$\sum_{n_2, n_1} P(n_1, n_2)\, \lambda_1^2(n_1 + 1)\, f_1^2(n_1 + 1, n_2)\, \lambda_2^2(n_2 + 1)$$

$$f_2^2(n_1 + 1, n_2 + 1)\, |A_2|^2 \tag{24b}$$

while $< S_{22}(t) >^u_\Delta$ is calculated from (16c) with (24a, 24b) substituted for $< S_{11} >$ and $< S_{22} >$.

It is seen from (23a, b) as well as $< S_{22} >^u_\Delta$ and $< S_{33} >^u_\Delta$ are never less than their initial values. This means that if the atom is initially in this state, and even with vacuum states for the two modes the system starts to develop. After some time t > 0 the other energy levels are populated.

D. Atom in a General State

Let the atom be in a state which is a mixed state with probability p_s for population of the level ω_s. Thus, the probability distribution function for the photons in the two modes for this case is given by :

$$P^m_\Delta(n_1, n_2, t) = \Sigma_s\, p_s\, P^s_\Delta(n_1, n_2, t)$$

where $P^3 = P^{gr}$, $P^2 = P^{in}$ and $P^{\pm} = P^u$ given by (12b, 17b, 21b).

Multiphon Processes

We turn our attention now to the multiphoton processes. We show in what follows that the model presented here can coupe with such processes. The Hamiltonian for this type is given by :

$$H = \sum_{i=1}^{3} \omega_i S_{ii} + \sum_{i=1}^{2} \Omega_i\, \hat{a}_i^+ \hat{a}_i + \sum_{j=1}^{2} \lambda_j (S_{j,j+1}\, \hat{a}_j^{m_j} + \hat{a}^{m_j^\dagger}\, S_{j+1} \tag{26}$$

This Hamiltonian can be transformed to the form of eq. (1) when we use the generalized boson operators $\hat{b}_i$ (*see:* Katriel and Hummer[8] and references there in) defined as follows :

$$\hat{a}_i^{m_i} = b_i \left\{ \frac{n_i}{[n_i/m_i]\,(n_i - m_i)\,!} \right\}^{\frac{1}{2}} = \hat{b}_i f_i(n_i)$$

$$\text{with } \hat{n}_{b_i} = \hat{b}_i^+ \hat{b}_i = [\frac{\hat{a}_i^+ \hat{a}_i}{m_i}] = [\frac{\hat{n}_i}{m_i}]$$

which relates the operation with $\hat{n}_{b_i}$ to $\hat{n}_i$

Using this transformation the Hamiltonian (26) takes the form

$$H = \sum_i \omega_i S_{ii} + \sum m_i \Omega_i b_i^{+} b_i + \sum_{j=1} \lambda_j (S_{j,j+1} \hat{b}_j f_j(\hat{n}_j) + f(\hat{n}_j) \hat{b}_j^{+} S_{j+1,j}) \tag{28}$$

which is the same form of (1) but with the functions f_j are functions of n_j only defined by (27a). Under these transformations and with (27b) we find in this case that :

$$\nu_0(n_1, n_2) = \lambda_1^2 \frac{n_1!}{(n-m_1)!} + \lambda_2^2 (\frac{n_2!}{(n_2-m_2)!})$$

$$\nu_1(n_1, n_2) = \nu_0(n_1, n_2 + m_2)$$

and

$$\nu_2(n_1, n_2) = \nu_0(n_1 + m_1, n_2 + m_2) \tag{29}$$

While for example for the ground state, we find that the probability of (12b) takes the form :

$$P_\Delta^{gr}(n_1, n_2, t) = \lambda_1^2 \lambda_2^2 (\frac{(n_1+m_1)!\ (n_2+m_2)!}{n_1!\ n_2!}) |A_2|^2 P(n_1 + m_1, n_2 + m_2)$$

$$+ \lambda_2^2 (\frac{(n_2+m_2)}{n_2!}) \frac{\sin^2 \mu_1 t}{\mu_1^2} P(n \quad n_2 + m_2)$$

$$+ | [e^{\frac{i\Delta t}{2}} + \lambda_2^2 (\frac{n_2!}{(n_2-m_2)!}) A_0] |^2 P(n \quad n_2) \tag{30}$$

where the A's and μ's are defined as before in (9a, b) with the ν's given by (29). With this example in mind, we can find the rest of the quantities given before.

Thus, with the model presented here, the multiphoton processes are discussed. The model contains detunning through the parameter Δ. When we write $\Delta = 0$ and f_i = constants, with $\lambda_1 = \lambda_2$ and $\omega_1 - \omega_2 = \omega_2 - \omega_3$ the model goes to the model discussed by Sukumar and Buck[6].

Discussion and Conclusions

In this article, we have presented a model for a 3-level atom in the "cascade" configuration. The interaction term depends on functions of the photon numbers. Under the condition (3) which introduces the parameter Δ, the Hamiltonian is exactly solvable. We have obtained the characteristic functions and given the time development for the expectation values of the photon numbers and the occupation numbers in the atomic levels. Also, the expectation values for n_1^2 and n_2^2 as well as $n_1 n_2$ are given for the different initial states of the atom. From these quantities, bunching and antibunching can be discussed for each mode through the functions $g_i(t) = (< n_i^2(t) > - < n_i(t) >)(< n_i(t) >)^{-2}$, $(i = 1, 2)$, especially their dependence on Δ. With this model, one is capable of discussing multiphoton processes by specifying the functions $f_i(n_1, n_2)$.

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الشكل السلمي لتفاعل مجموعة ذات ثلاثـــة مستويات ونمطين للمجال

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نقـدم في هذا البحث نموذجاً للتفاعل بين نمطين من أنماط المجال ومجموعة ذات ثلاثة مستويات للطاقة في الشكل السلمي . في هذه الدراسة ، يوجد بارامتر لعدم الانضباط ، ولقـد تم حل النمـوذج في حالـة وجود دوال لعدد الفوتونات في نمطى المجال في الدالة الهاملتونية للمجموعة . ثم حساب دالة التوزيع الاحتمالية والدوال المميزة والقيم المتوسطة لعدد الفوتونات في كل نمط وأعداد الاحتلال في المستويات المختلفة . ولقد أمكن دراسة التفاعلات التي تعتمد على عديد من الفوتونات وذلك باختيار خاص لدوال العدد .